Mathematics: Analysis & Approaches SL

Formula Sheet - First examination 2021

Chapter 1: From patterns to generalizations: Sequences and series

A sequence is a list of numbers written in a defined order, ascending or descending, following a specific rule.

A finite sequence has a finite number of terms.
An infinite sequence has an infinite number of terms.

A formula or expression that mathematically describes the pattern of the sequence can be found for the general term, u_n .

A recursive sequence uses the previous term or terms to find the next term. The general term will include the notation u_{n-1} , which means the "previous term".

A sequence is called **arithmetic** when the same value is added to each term to get the next term.

Sigma notation	$$\sum_{n=1}^{5} 3n-2$$ 5: is the upper limit of the series 1: is the lower limit of this series "n": is called the index and represents a variable. The values of n will be consecutive integers. $3n-2$: is the general term of this series.
The nth term of an **arithmetic sequence**	$$u_n = u_1 + (n-1)d$$ u_1: is the first term n: is the position of the term in the sequence. d: common difference
	$$S_n = \frac{n}{2}(2u_1 + (n-1)d)$$

Sum of n terms of an **arithmetic sequence** is called an arithmetic series	$S_n = \frac{n}{2}(u_1 + u_n)$

A sequence is called **geometric** when each term is multiplied by the same value to get the next term.

The nth term of a **geometric sequence**	$u_n = u_1 r^{n-1}$ u_1: is the first term. n: is the position of the term in the sequence. r: common ratio.
Sum of n terms of a **geometric sequence** is called a geometric series.	$S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, \quad r \neq 1$
A diverging sequence is when	$r < -1$ or $r > 1$
A converging sequence is when	$-1 < r < 1$

If $r = 1$ then you will have a constant sequence, not a progression as all the terms will be the same.

The sum of an infinite geometric sequence	$S_n = \frac{u_1}{1 - r}, \quad \lvert r \rvert < 1$
Simple interest: A is the accumulated amount, P is the principle, r is the annual rate, n is the time in years	$A = P(1 + nr)$
Compound interest: A is the final amount, P is the principle, r is the annual interest, n is the number of compoundings in a year, t is the total number of years.	$A = P\left(1 + \frac{r}{n}\right)^{nt}$ $\text{n} = 1$ when yearly, $\text{n} = 4$ when quarterly, $\text{n} = 12$ when monthly

Binomial theorem	$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$
Binomial coefficient	$\binom{n}{r} = nC_r = \frac{n!}{(n-r)!r!}$
$n!$ Is called n factorial and is calculated as	$n! = n(n-1)(n-2)(n-3)\ldots 3 \times 2 \times 1$

A mathematical proof is series of logical steps that show one side of a mathematical statement is equivalent to the other side for all values of the variable.

The goal of an algebraic proof is to transform one side of the mathematical statement until it looks exactly like the other side.
One rule is that you cannot move terms from one side to the other

At the end of the proof, we write a concluding statement, such as LHS ≡ RHS or QED