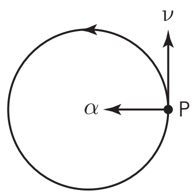
# Solutions for Topic 6 – Circular motion and gravitation

**1.**




**2.** friction between the tyres and the road.

**3.** $F = \frac{mv^2}{r}$.

**4. a) (i)** The drops are increasingly far apart and so the speed is increasing.

**(ii)** 5.6 s is 6 time intervals so the distance travelled is 14.4 cm on the scale, or 57.6 m on the ground.

**b) (i)** centripetal force; acting towards the centre of the circle

**(ii)** Passengers are in a rotating frame of reference. Seen from above the passengers would move in a straight line from Newton's First Law of motion but friction acts at seat to provide centripetal force to centre of circle. Passenger interprets the reaction to this force as being flung outwards.

**5. a)** Circumference is $2\pi \times 85 = 534$ m. So linear speed is $\frac{534}{30 \times 60} = 0.297 \text{ m s}^{-1}$.

**b) (i)** change in weight $= m\frac{v^2}{r}$ so fractional change $= \frac{m\frac{v^2}{r}}{mg} = \frac{v^2}{gr} = \frac{0.59^2}{9.8 \times 170} = 2.1 \times 10^{-4}$

**(ii)** a smaller apparent weight as in passenger frame of reference there is an apparent additional upward force

**c)** Capsule turns $2\pi$ rad in 30 minutes, so 3.5 mrad $\text{s}^{-1}$.

**6.** Angular speed of Earth $= \frac{2\pi}{24 \times 60 \times 60} = 7.3 \times 10^{-5} \text{ rad s}^{-1}$

Linear speed $v = \omega r \cos\theta$ where $\theta$ is the latitude.

**a)** 14′ of arc = 4.1 mrad, linear speed $= 7.3 \times 10^{-5} \times 6.4 \times 10^{6} \times 1 = 470 \text{ m s}^{-1}$

**b)** 46° gives 320 $\text{m s}^{-1}$

**c)** At the geographical south pole the linear speed is zero.

**7. a)** Maximum friction force $= 6500 \times 9.8 \times 0.7 = 44.6$ kN

Centripetal force required $= m\frac{v^2}{r} = \frac{6500}{150}v^2$. So $v_{max} = \sqrt{\frac{44600 \times 150}{6500}} = 32 \text{ m s}^{-1}$

**b)** $m\frac{v^2}{r} < mg$; $v > \sqrt{rg}$. Maximum speed $= \sqrt{75 \times 9.8} = 27 \text{ m s}^{-1}$

**8.** The component of the normal reaction force acting horizontally contributes to the centripetal force so the faster the cyclist is travelling, the greater the component required and this is achieved by moving up the slope to a point where the slope angle is greater.

**9.** Gravitational force on planet provides the centripetal force on the planet (Keplar's third law)

so $m_p \omega^2 R = G\frac{m_S m_p}{R^2}$

re-arranging $\omega^2 = G\frac{m_S}{R^3}$

$\omega = 2\pi f = \frac{2\pi}{T}$

and $T^2 = \frac{4\pi^2 R^3}{Gm_s}$



**10. a)** Speed is a scalar but velocity is a vector. The direction of the velocity is constantly changing, so the vector velocity is changing too. Acceleration occurs when velocity changes and so there is acceleration in this case.

**b)** Work done = distance travelled × force in direction of distance travelled. The force (acceleration) acting and the distance travelled are at 90° to each other so in this case no work is done. ***OR***
Work is done when kinetic energy or potential energy change. The speed is constant so kinetic energy is constant. Distance from Earth is constant so gravitational potential energy does not change. So no work is done.

**11.** $g = \frac{Gm}{r^2}$

$$\frac{Gm_E}{r_E^{\ 2}} = \frac{Gm_M}{r_M^{\ 2}}$$

Where $r_E$ and $r_M$ are the distances from the centres of Earth and Moon respectively to the point where the field strengths are equal (known as the Lagrangian point).

So $\frac{r_E^{\ 2}}{r_M^{\ 2}} = \frac{m_E}{m_M} = \frac{6 \times 10^{24}}{7.3 \times 10^{22}} = 82$

$\frac{r_E}{r_M} = \sqrt{82} = 9.06$

Therefore the point is $\frac{9}{10}$th of the way from the Earth to the Moon ($3.42 \times 10^8$ m).

**12. a) (i)** force on 1 kg of water $= \frac{G\,m_M}{r^2} = 3.4 \times 10^{-5}$ N due to Moon

**(ii)** force on 1 kg of water $= \frac{G\,m_S}{r^2} = 6.0 \times 10^{-3}$ N due to Sun

**b)** When the Moon is overhead there is a gravitational force of attraction (a tide) on objects. So fluids are able to respond to this by an increase in the water level (tides are also observed in the rocks). There are two tides because there is a corresponding bulge in the surface on the opposite side of the Earth.

**13. a)** $g = -\frac{GM}{r^2} = \frac{6.7 \times 10^{-11} \times 6 \times 10^{24}}{(7.4 \times 10^6)^2} = 7.3 \text{ N kg}^{-1}$

**b)** The satellite orbits in 24 hours; orbital time = 86400 s. Angular speed $= \frac{2\pi}{86400} =$ $7.3 \times 10^{-5}$ rad s$^{-1}$

**c)** $\frac{mv^2}{r} = \frac{GM_E\,m}{r^2} = \frac{6.7 \times 10^{-11} \times 6 \times 10^{24} \times 1.8 \times 10^3}{(3.6 \times 10^7)^2} = 560$ N